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Observations on the paper entitled "On the Heptic equation with five unknowns $x^4+y^4-(y+x)w^3=14z^2T^5$"

J.Shanthi[1], S. Aarthy Thangam[2], S.Vidhyalakshmi[3], M.A.Gopalan[4]

[1,2,3]Assistant Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.

[4] Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.

Abstract:

In this paper, we obtain infinitely many non-zero integer quintuples (x, y, z, w, T) satisfying the non-homogeneous equation of degree seven with five unknowns given by $x^4+y^4-(y+x)w^3=14z^2T^5$.

Introduction:

It is known that higher degree Diophantine equations with multiple variables are rich in variety. While searching for the collection of seventh degree Diophantine equations with five unknowns, the authors came across the paper [1] entitled "On The Heptic Equation with five unknowns $x^4+y^4-(y+x)w^3=14z^2T^5$"

In the above paper, the authors have presented only a few choices of integer solutions. However, there are many more sets of integer solutions to the considered equation which is the main thrust of this paper.

Method of analysis:

The Diophantine equation representing the non-homogeneous equation of Degree seven with five unknowns is given by

$$x^4+y^4-(y+x)w^3=14z^2T^5 \tag{1}$$

Introduction of the transformations

$$x=w+z,\quad y=w-z, w\neq z \tag{2}$$

in (1) leads to

$$z^2+6w^2=7T^5 \tag{3}$$

It is observed that (3) is satisfied by

$$z=7^3 m(m^2+6n^2)^2, w=7^3 n(m^2+6n^2)^2, T=7(m^2+6n^2) \quad (4)$$

In view of (2), one has

$$x=7^3(n+m)(m^2+6n^2)^2, y=7^3(n-m)(m^2+6n^2)^2, n\neq m \quad (5)$$

Thus, (4) and (5) represent the integer solutions to (1).

It is to be noted that (3) may be solved through different ways leading to different sets of integer solutions to (1).

Way 1:

Considering

$$z=\alpha T^2, w=\beta T^2, \alpha\neq\beta \quad (6)$$

in (3), it is written as

$$\alpha^2+6\beta^2=7T \quad (7)$$

which is satisfied by

$$\alpha=a-6b, \beta=a+b \quad (8)$$

$$T=a^2+6b^2 \quad (9)$$

In view of (6) and (2), we have

$$\left.\begin{array}{l} z=(a-6b)(a^2+6b^2)^2, w=(a+b)(a^2+6b^2)^2, \\ x=(2a-5b)(a^2+6b^2)^2, y=7b(a^2+6b^2)^2 \end{array}\right\} \quad (10)$$

Thus, (9) and (10) represent the integer solutions to (1).

Note: 1

Write 7 as

$$7=\frac{(13+i\sqrt{6})(13-i\sqrt{6})}{25} \quad (11)$$

Using (9), (11) in (7) and employing the method of factorization, one obtains after some algebra, the following two sets of integer solutions to (1):

Set 1:

$$z=(25)^2(13a-6b)(a^2+6b^2)^2, w=(25)^2(a+13b)(a^2+6b^2)^2,$$
$$x=(25)^2(14a+7b)(a^2+6b^2)^2, y=(25)^2(-12a+19b)(a^2+6b^2)^2,$$
$$T=25(a^2+6b^2)$$

Set 2:

$$z=(13k+4s-13)[(5k+2s-5)^2+6s^2]^2, w=(k+3s-1)[(5k+2s-5)^2+6s^2]^2,$$
$$x=(14k+7s-14)[(5k+2s-5)^2+6s^2]^2, y=(-12k-s+12)[(5k+2s-5)^2+6s^2]^2,$$
$$T=(5k+2s-5)^2+6s^2$$

Way II:

Considering

$$z=\alpha T, w=\beta T, \alpha\neq\beta \tag{12}$$

in (3), it is written as

$$\alpha^2+6\beta^2=7T^3 \tag{13}$$

which is satisfied by

$$\alpha=49m(m^2+6n^2), \beta=49n(m^2+6n^2)$$
$$T=7(m^2+6n^2) \tag{14}$$

In view of (12) and (2), we have

$$\left.\begin{aligned} z&=7^3 m(m^2+6n^2)^2, w=7^3 n(m^2+6n^2)^2,\\ x&=7^3(n+m)(m^2+6n^2)^2, y=7^3(n-m)(m^2+6n^2)^2,\end{aligned}\right\} \tag{15}$$

Thus, (14) and (15) represent the integer solutions to (1).

Note: 2

Write 7 on the R.H.S. of (13) as

$$7=\frac{(13+i\sqrt{6})(13-i\sqrt{6})}{25} \tag{16}$$

Using (9), (16) in (13) and employing the method of factorization, one obtains, after some algebra, the following set of integer solutions to (1):

$$z=25^2(a^2+6b^2)[13(a^3-18ab^2)-6(3a^2b-6b^3)],$$
$$w=25^2(a^2+6b^2)[(a^3-18ab^2)+13(3a^2b-6b^3)],$$
$$x=25^2(a^2+6b^2)[14(a^3-18ab^2)+7(3a^2b-6b^3)],$$
$$y=25^2(a^2+6b^2)[-12(a^3-18ab^2)+19(3a^2b-6b^3)],$$
$$T=25(a^2+6b^2)$$

Remark:

One may also write 7 as

$$7=(1+i\sqrt{6})(1-i\sqrt{6})$$

In this case, the corresponding integer solutions to (1) are given by

$$z=(a^2+6b^2)(a^3-18ab^2-18a^2b+36b^3),$$
$$w=(a^2+6b^2)(a^3-18ab^2+3a^2b-6b^3),$$
$$x=(a^2+6b^2)(2a^3-36ab^2-15a^2b+30b^3),$$
$$w=(a^2+6b^2)(21a^2b-42b^3),$$
$$T=(a^2+6b^2)$$

Note 3:

The choice

$$T=\beta$$

in (13) gives

$$\alpha^2=\beta^2(7\beta-6)$$

which is satisfied by

(i) $\alpha=(7k-1)(7k^2-2k+1),\beta=(7k^2-2k+1)$

(ii) $\alpha=(7k+1)(7k^2+2k+1),\beta=(7k^2+2k+1)$

After performing some algebra, the corresponding sets of integer solutions to (1) are as follows:

Solutions from (i):

$$z=(7k-1)(7k^2-2k+1)^2,w=(7k^2-2k+1)^2,T=(7k^2-2k+1),$$
$$x=7k(7k^2-2k+1)^2,y=(2-7k)(7k^2-2k+1)^2$$

Solutions from (ii):

$$z=(7k+1)(7k^2+2k+1)^2,w=(7k^2+2k+1)^2,T=(7k^2+2k+1),$$
$$x=(7k+2)(7k^2-2k+1)^2,y=-7k(7k^2-2k+1)^2$$

Note 4:

The choice

$$T=\alpha$$

in (13) gives

$$6\beta^2=\alpha^2(7\alpha-1)$$

which is satisfied by

(i) $\beta=(7k-6)(42k^2-72k+31),\alpha=(42k^2-72k+31)$

(ii) $\beta=(7k-1)(42k^2-12k+1),\alpha=(42k^2-12k+1)$

After performing some algebra, the corresponding sets of integer solutions to (1) are as follows:

Solutions from (i):

$$w=(7k-6)(42k^2-72k+31)^2, z=(42k^2-72k+31)^2, T=(42k^2-72k+31),$$
$$x=(7k-5)(42k^2-72k+31)^2, y=(7k-7)(42k^2-72k+31)^2$$

Solutions from (ii):

$$w=(7k-1)(42k^2-12k+1)^2, z=(42k^2-12k+1)^2, T=(42k^2-12k+1),$$
$$x=(7k)(42k^2-12k+1)^2, y=(7k-2)(42k^2-12k+1)^2$$

Conclusion:

In this paper, we have obtained different patterns of solutions to Heptic equation with five unknowns. As Diophantine equations are rich in variety, one may search for other choices of Heptic equations with multi variables for obtaining their integer solutions.

Reference:

[1] S. Vidhyalakshmi, M.A. Gopalan, S. Aarthy Thangam, On the Heptic equation with five unknowns $x^4+y^4-(y+x)w^3=14z^2T^5$, IJESRT, 8(1), Pp 137-140, January 2019.